Experiments and Theories, A Fundamental Model

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Abstract

A fundamental model of experiments and their relationship to theories is offered. The physical nature of the model should be seen as a reflection of the epistemic *and* ontic nature of physics required by quantum theory, as well as the causal models of general relativistic space-times. By encoding experiments as Monads, we see them as physical aspects of nature. By the Adjoint Functor Theorem, theories are bound to experiments in a way that is deeply meaningful, and essentially mechanical.

1 Introduction

Modern physics faces a dilemma that has been around for a century, but has never been fully resolved. In particular, because Quantum Theory is essentially a probability theory, the concept of information is deeply embedded in physics in ways that are, arguably, not understood well at all. This dilemma has been inspected in great detail in recent years [7] [8], and quantum mechanics has seen attempts at derivation from various axiomatic arguments [4] [5]. Something that has not been probed properly is the fact that experiments are, themselves, both epistemic and ontic events. During an experiment, we express our present beliefs, gain data, and then update our belief about a theory. It is likely an effect of the desire for realism that experiments are treated as entirely subjective and not worthy of a role in the central theory of physics. This need not be so, if we express, explicitly and in general, what experiments are.

The program of quantum gravity is conceptually exhilarating. The depths to which one must explore in order to have even an inkling of such a theory are breathtaking and are not for everyone. This paper takes some of the approaches to quantum gravity, including work in categorical quantum mechanics and the causal structure approach [2][3] [1] [6]. It attempts to distill them into a fundamental theory that would work to do something arguably more general. In fact, this paper should outline a deep theory that could form a foundation for all of physics and also for science. A theory that gives a physical derivation of experiments themselves, should put physics where it belongs, into the deepest possible foundation for all of science. This theory may also provide a way to have a physics at all epochs of a universe, from today to the first event in an observer's causal history.

2 Experiments

There are several steps in performing an experiment. First, you must have a theory that you wish to test, which you must acknowledge. Next, you need to design and build an experimental apparatus to carry out the experiment. Once, you have your apparatus, you interact with it. After you have interacted with the apparatus, you then update your belief about your theory. In this paper, we will only speak of having an apparatus and a theory and also about interacting with the apparatus.

In general, you carry out an experiment by changing an aspect to your apparatus and observe the changes in a separate aspect of your apparatus.

For instance, if you wanted to show that the acceleration due to gravity is independent of the mass of an object, you would do the following:

1. design a means of varying the mass of an object (Perhaps by having different objects of varying mass)
2. design a way to measure the time it takes for an object to fall a fixed distance
3. vary the mass of the object and record the time of falling for each mass

In the end, you would produce a table that would show that the time for the object to fall is not a function of the mass of the object. This sounds like a perfectly acceptable outline for an experiment, however there are a few subtleties missing. For instance, given the chance to measure the fall-time of an object, a good experimenter would never do this measurement once. They would do it several times. This would produce a list of random numbers centered around a mean time for each mass. A better experiment would be as follows:

1. design a means of varying the mass of an object
2. design a way to measure the time it takes for an object to fall a fixed distance
3. choose a mass and measure the fall-time for that mass several times
4. vary the mass of the object and re-run the experiment, measuring the fall-time several times for each mass

There are a few things one can say already. First, every experiment consists of an interface we call a measurement device, that gives us access to some aspect of a system under study. In this case, the interface would be the weight scale and its readout and the clock used to time the fall and its readout and also the buttons on the clock to start and stop the clock. Naturally, there is also a system under study. In this case, the system under study is the force of gravity, space-time, and some instances of massive objects. The mass scale gives us access to the mass of the objects.

3 Category Theory

Category theory is a branch of mathematics that uses transformations, called morphisms and functors, to encode axiomatic data about structure. That is a gross oversimplification and would still likely deter most physicists from using Category Theory. Another way to understand category theory is to understand that, for every type of mathematical structure, there is a corresponding type of transformation that preserves this structure. For instance, in their most abstract sense, Sets have associated functions that transform sets into yet more sets, thus preserving the structure of the object upon which they act (ie Set structure). Groups have group-homomorphisms that preserve group structure and map groups to yet more groups(preserving group structure). Vector spaces have general linear transformations that take vector spaces and produce yet more vector spaces. One can go on with this and apply this to every aspect of mathematics. Category theory is sometimes considered, along with Set Theory, as a foundation for mathematics itself.

Each category is made up of the structured objects which it concerns as well as all the morphisms that map objects to objects. It should be pointed out that the data that makes up a category is not the details of the structured objects themselves. Instead, the data of a category is actually the algebraic relations between morphisms and their composites. An example of a category is SET, the category whose objects are sets and whose morphisms are functions. Categories can be finite, and in that case, they consist of a finite number of objects, a finite number of morphisms and a finite number of equations over words formed from composites of morphisms.

In the case of experiments, one might have already noticed that when we change a feature of our experiment, we preserve the experiment in a precise sense. In other words, as we change the mass of an object and measure its mass, we always end up with what we started with, namely a massive object and a mass scale. For instance, our experiment would end if we broke the mass scale. Thus, the interface of the apparatus is like a category and has associated transformations that preserve the structure of the interface.

In the same way that functions are transformations that preserve Set structure, there are transformations that preserve Category structure and map categories to yet more categories. These transformations are called functors. As an example, there is a functor that maps the category of Groups to the category of Sets. It does this by taking a group and returning the underlying set of that group, ie the set of group elements. This is known as the forgetful functor, and is called this because it "forgets" the added structure of the group and returns only the elements. It takes one category and maps it to another category, preserving category structure. Slightly more precisely, functors take the objects of a category and map them to the objects of another category. They also take the morphisms of a category and map them to the morphisms of another category. Every functor will take an identity morphism in one category and map it to an identity morphism in another category.

The arrow, colon and dot ($\rightarrow$, :, $\cdot$) are essential syntax for Category theory. For instance, a function, f, that maps a set, A, to a set, B can be expressed in the following way:

$$f : A \rightarrow B.$$

A functor, F that maps a category, $\mathcal{A}$, to a category $\mathcal{B}$ can be expressed in the same way:

$$F : \mathcal{A} \rightarrow \mathcal{B}.$$

Two morphisms can be composed if the source of one is the target of the other and that looks like this:

$$f : A \rightarrow B,$$

$$g : B \rightarrow C,$$

$$g \cdot f = h.$$

An identity morphism, e, works like multiplying by 1.

$$e : A \rightarrow A$$

and can be composed with any morphism, f.

$$e \cdot f = f = f \cdot e$$

One can map a category to itself, in that the objects of the category are mapped to objects of the same category and likewise with the morphisms. This is called an endofunctor. One special endofunctor is the identity that maps every object to itself and every morphism to itself. Endofunctors form the basis of what are called Monads which we discuss in the next section.

3.1 Monads

Because an endofunctor maps a category to itself, one can always compose an endofunctor with itself. That is, you can apply the endofunctor any number of times because your output is the same as your input, namely category $\mathcal{C}$. For instance, this is always valid,

$$F \cdot F \cdot F \ldots,$$

and causes a string of transformations like this,

$$\mathcal{C} \rightarrow \mathcal{C} \rightarrow \mathcal{C} \rightarrow \mathcal{C} \ldots$$

for every endofunctor, F, of category $\mathcal{C}$. Given an endofunctor F, one may map this endofunctor to yet another endofunctor $\upsilon : F \rightarrow G$. This is called a natural transformation. The identity on category $\mathcal{C}$ is $1_{\mathcal{C}}$. These facts combine to produce the following possibilities:

$$\mu : F \cdot F \rightarrow F$$

$$\eta : 1_{\mathcal{C}} \rightarrow F$$

If you can find two such natural transformations, μ, η for F, then you have what is called a monad. The monad is the combination of the endofunctor and the natural transformations, and is given as (F, μ, η).

3.1.1 The List Monad

What are lists? They are spelled out perfectly in [10] and I will describe them here. One simple model of a lists dictates that you must have an alphabet, or set of symbols, or set of objects that form the elements of the lists. Lets call this set Σ. Then a list is a string made up of elements of the set. For every set, Σ, there is a set of lists. To go from your set of objects to your set of lists, we use a functor that maps Set to Set. That is to say, we have a functor $F : SET \to SET$ such that we take every set and map it to its set of lists over that set. That is the endofunctor of our monad. Next, we need to find the following natural transformation $\mu : F \cdot F \to F$, in that it has to take two applications of the list functor and turn it into one application of the list functor. So, what does $F \cdot F$ look like? Well, it takes a set of lists to a set of "lists of lists". An example might be useful now.

Let's say that our set of objects, Σ is $\{a, b, c\}$, then our set of lists has elements

$$\{[], [a], [b], [c], [ab], [aaa], [abbbc], \ldots\}$$

We apply F again on this set to get a list of lists that look like the following :

$$\{[], [[a], [b], [aaa]], [[c], [cbbba], [], [bbca]], [[cbbbbab], \ldots], \ldots\}$$

The natural transformation μ takes this set of lists of lists and returns just a set of lists by concatenating. So, for example, we could have the following

$$[[a], [b], [aaa]] \to [abaaa]$$

Next, we have to find the following natural transformation $\eta : 1_C \to F$ and this corresponds to mapping a set to a list of singletons, so,

$$\eta : \{a, b, c\} \to \{[a], [b], [c]\}$$

4 Experiments, Categorically

4.1 Lead as Radiation Shielding

A simple example of an experiment is one that demonstrates the power of lead as a shield against radioactive particles. To quantify how lead can stop Alpha particles, we can design an apparatus that has the following pieces:

1. A source of alpha particles: a small lump of radioactive material
2. A collection of lead plates of thickness, t.
3. A gieger counter

The apparatus is arranged as in fig 1.

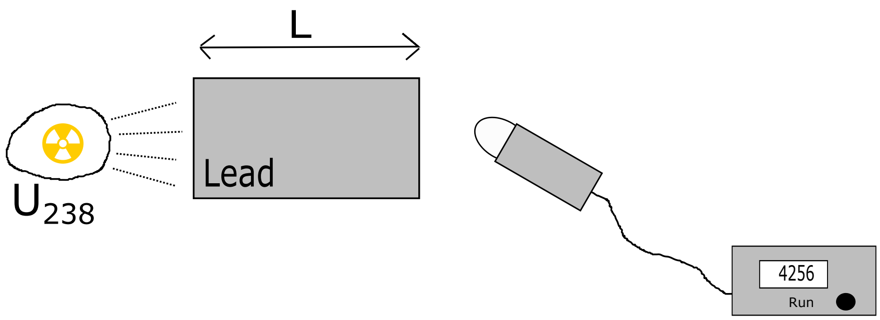

Figure 1: A simple experiment to test the stopping power of lead for radiation

The Geiger counter has a very simple interface, it is just a readout that prints an integer and a run button. We understand the integer to mean the number of radioactive alpha particles that hit the Geiger counter in one second. The run button clears the counter, and then starts the counter, allowing it to run for exactly one second. We perform the experiment in the following way.

1. Place the radioactive lump a short distance away from the Geiger counter (say two feet).
2. Press the run button on the Geiger counter and read the number, being sure to record it.
3. Repeat step 2 many times, say four hundred times.
4. Add one piece of lead into the path between the Geiger counter and the radioactive lump. Record this.
5. Run the Geiger counter many times. Each time, record both the thickness of the lead, or number of lead plates, and the number from the counter
6. Repeat steps 4 and 5 for many plates of lead, say thirty.

The apparatus has two numbers that dictate the state of the apparatus, namely the number of lead plates (or their total thickness) and the number on the Geiger counter. A typical record might be (5, 3432), which indicates five plates and three thousand four hundred and thirty two alpha particles detected in one second. Two records looks like the following [(7,432),(0,64)] and four records looks like the following [(6,743),(19,354),(12,65), (3, 82)]. It should be clear that any attempt at an experiment will be a list of records.

One property of scientific experiments is that many different people, working in different laboratories, must be able to perform the same experiment. If two different people have done the same experiment, we have a list of two lists, like this [[(6,743),(19,354)], [(12,65), (3, 82)]]. It is a fundamental property of science that we should be able to combine these two lists to produce one list, one monolith of data:

$$[[(6,743),(19,354)],[(12,65),(3,82)]] \to [(6,743),(19,354),(12,65),(3,82)] \quad (1)$$

This explains how reproducing science experiments goes towards supporting a single theory.

We have seen two properties of the List monad. First is that lists are made from sets, and second, that multiple experiments (encoded as the list endofunctor applied twice) can be concatenated into one large experiment. That is, we have defined our endofunctor F and found the meaning of the natural transformation, $\mu : F \cdot F \to F$. What about the other natural transformation, $\eta : 1_C \to F$? The existence of this natural transformation implies that we know what the states of the apparatus are and can write them down immediately. The configurations of the apparatus are data too.

5 A Fundamental Theory for Physics and Science in General

In science, and most prominently in physics, we find two very important systems which are present every time we conduct an experiment or conceive of an experiment. First, there is a theory of our system under study, and speaking in a realist sense, there is an *actual* system which *exists* and it is the focus of our study. Next, we have our apparatus and, in general, we need to understand the interface of this apparatus. By 'The Interface' to the apparatus, I mean the buttons, knobs, dials and readouts of the apparatus. For instance, a rheostat, or variable resistor should have a simple interface involving a dial and numbers. The position of the dial points to a number indicating the resistance of the rheostat. We could just say that there is a number on the rheostat and leave it at that. The state of the rheostat would be given by that number. The dial allows us to change that number. The interface of the apparatus allows us to transform the apparatus. A simple, though powerful model of the apparatus is just to include only the transformations offered via the interface.

5.1 A Cosmological Experiment

A Galaxy is a collection of stars. We believe in the existence of galaxies because we have telescopes and we have theories of gravity that explain their form. These telescopes have allowed us to look into the sky and see things that are very far away, very old and very large. Experiments exist, arguably for one of two purposes. We use them either to establish a causal or correlative link between two random variables, or we use them improve our understanding of a theory. The current work should establish, in a mechanical way, the link between these two ways of viewing an experiment. In the case of this experiment, we are going to try to update our belief about a theory. In particular, let us suppose we have a theory about how galaxies form. The theory goes as follows: galaxies form by the collapse of large gas clouds. We believe that a galaxy starts out as a very large, amorphous gas cloud that collapses and as it collapses, it forms a spiral pattern and becomes a spiral galaxy. How do we prove this theory?

We are trying to probe our theory of how galaxies form. This means that we are interested in probing the ways galaxies evolve. We can picture the evolution of a galaxy as in figure 2.

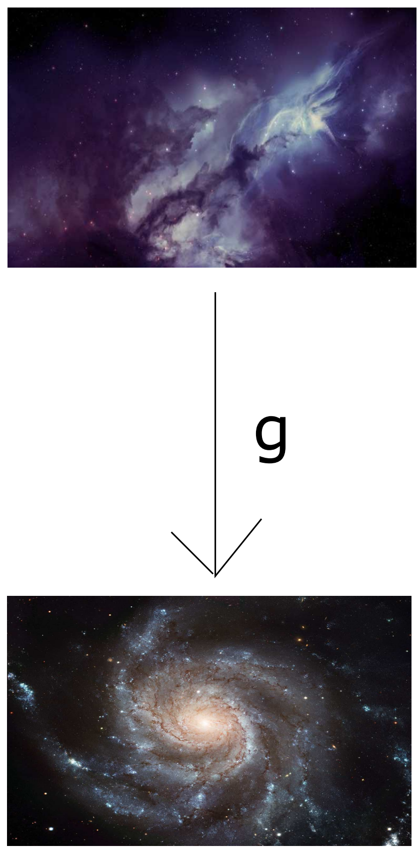

Figure 2: An amorphous gas cloud becomes a spiral galaxy

This encodes the theory we have about the evolution of galaxies. A galaxy is an object in a category, a very specific category, and its evolution to a new state is encoded in a very specific map, g. All the specific details are unimportant for our purposes, but it is certain that categories can encode a tremendous amount of information. Given any theory for what a galaxy is, we can encode it in a category. This is what it means to "have a theory".

Before we describe the experiment that will help us support our theory about the evolution of galaxies, let's examine the apparatus and how we must interact with it. Naturally, the apparatus we use to learn about distant galaxies is a telescope. See figure 3. In the case of a telescope, we have three basic transformations we can make of our apparatus. We can rotate the axis T_1. We can rotate the axis T_2, and we can turn the focus knob, f. In each case, after applying the transformation we will see a change in the eyepiece and we can call a transformation of the eyepiece e. It may not immediately make sense that the eyepiece is changing. Consider the eyepiece as being a photo-diode array like the ones found in CCD cameras. We could also consider the eyepiece as being a pixel array. Each pixel is, for arguments sake, one bit that is either 0 or 1, or black and white. In this way, it is easier for us to say that the eyepiece has a state and it changes when the image changes. The structure preserving maps of the telescope are T_1, T_2, f, and e. If we were to take the eyepiece out and grind it into dust, we would no longer have a telescope and as such, we don't consider that kind of map to be an aspect to the telescope system.

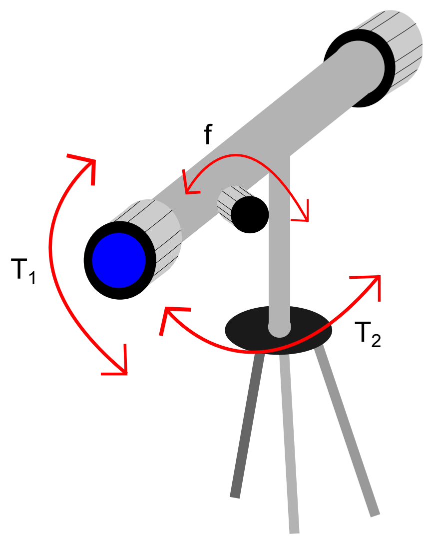

Figure 3: A telescope is a limited interface to distant galaxies

Now let us consider an experiment. Through this experiment, we want to prove the following theory : Spiral Galaxies are formed from the collapse of monolithic gas clouds. This is a theory of galactic structure and states that galaxies are formed from large amorphous gas clouds which collapse and these mechanics lead to the formation of spiral galaxies. How do we use our telescope to prove this?

First, we have to know that the further we look into space, the older the objects are that we can focus on. That is, as we change our focus knob, we are looking back into older and older parts of the universe. With this fact in hand, we may now devise an experiment.

1. Train our focus knob on deep space.
2. Use the angles to see many different structures.
3. Record each structure and their distance and label them as either amorphous gas clouds or spiral galaxies.
4. Compute the proportion of structures that are spiral galaxies.
5. Rotate the focus knob to look at objects that are closer to us, and thus newer. Record all the structures and their labels.
6. Compute the proportion of spiral galaxies for each focus knob setting.
7. Repeat steps 5 and 6 for many focus settings.

An endofunctor on the category that describes our telescope maps transformations of the telescope to transformations of the telescope. We are interested in the functor that maps the morphism f, turning of the focus knob, to the changing of the eyepiece e. In Figure 4, we demonstrate what this looks like.

This is the endofunctor on our local lab.

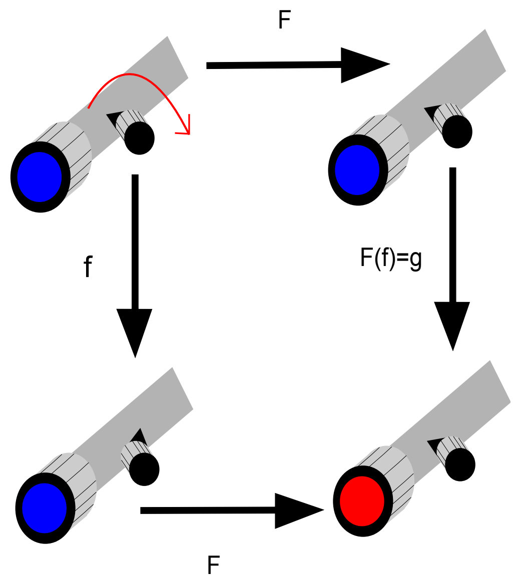

Figure 4: Basic interaction with a telescope: turning the knob f changes the view through the eyepiece g

Next, we want to understand that we probed the evolution of the distant galaxy using morphisms in our local lab. How did we do that? We changed our focus knob. Recall that as we turned the focus knob, we were looking at things closer to us in time. Conceptually, we understood this as a probe of "how the galaxy changes". We can picture this process too as in figure 5.

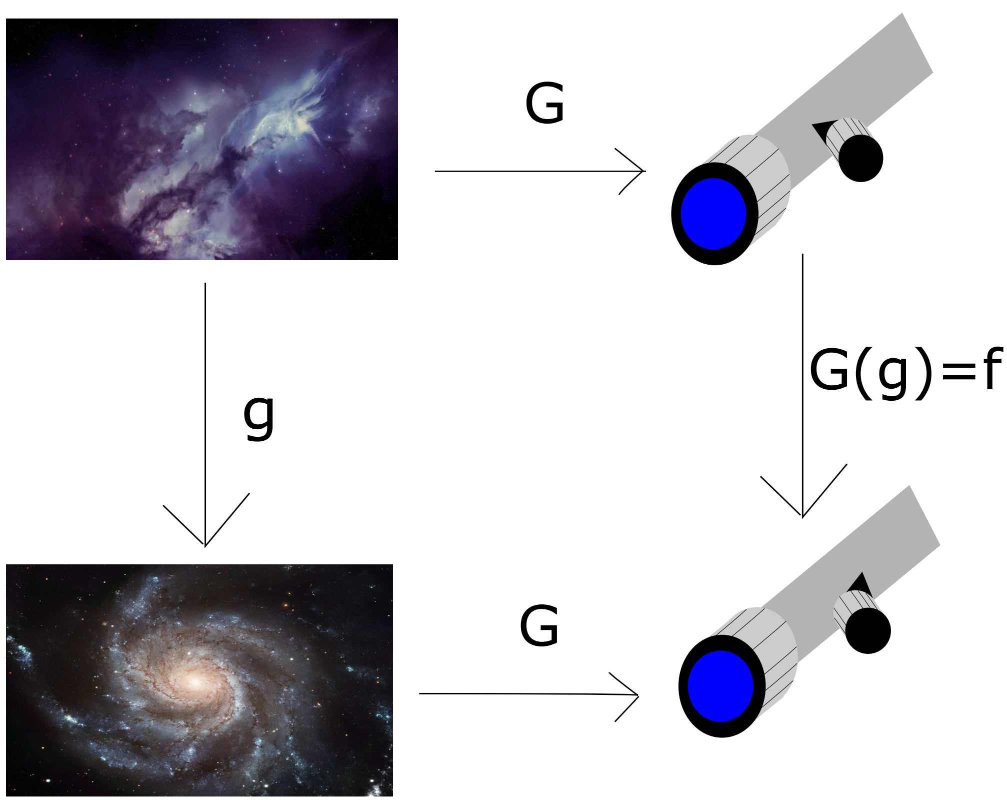

Figure 5: The structure preserving transformation of the galaxy is mapped into the structure preserving transformation of our local telescope

In figure 5, we made a conceptual connection. We mapped the transformation of the galaxy into a transformation in our local lab, namely into the turning of the focus knob. One precise way to map arrows in a category into arrows in another category is, as we have seen, a functor. Thus, we have a functor G that maps a distantly understood system, namely a galaxy, into a local system, namely a telescope. Each system is understood as a category with structure preserving maps.

The experiment that we did with the telescope is very much like our experiment with the radioactive material and the lead shielding. It also conforms to a List Monad. We have the local Monad that expresses what happened in our local lab. We also have a functor from a category that describes the theory for the system under study that goes into the system we used to perform the experiment. In the next section, we will discuss a general theory that states that the experiment in our example is related to the theory in our example via the Adjoint Functor Theorem.

A map from a causal system into a new causal system has a deep root in general relativity. Several approaches to quantum gravity encode space-times as causal structures and this is especially evident in [6]. According to [6], one can see a universe as a domain, which is an ordered structure. A structure preserving map of a space-time, therefore, is a domain map. This kind of map is pictured in figure 6.

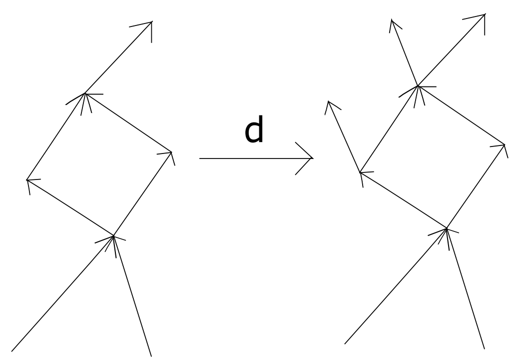

Figure 6: A spacetime map, d, is a domain map. It's also a functor.

What we should notice in figure 6, is that a domain is an ordered structure. This means that a map from a domain to a domain is a map from a diagram to a diagram, namely it is a functor. This indicates that the idea of mapping causal diagrams of a distant (older) system into a causal diagram of a local (recent) system is deeply embedded in general relativity and is thus physical. This gives us some confidence that experiments are not entirely subjective, but are aspects of the evolution of our universe. An experiment is intimately linked to the evolutionary mechanics of the universe. Furthermore, seen this way, the concept of information becomes embedded in physics in a natural way.

5.2 Theories and Experiments

Recall that the reason we are interested in the telescope experiment is that we want to test a theory about how galaxies evolve. We believe that there is a structure preserving map that maps a galaxy to a galaxy and, in particular, we are interested in the map, g that transforms amorphous gas clouds to spiral galaxies. When we perform an experiment such as this, we are mapping g, which describes the distant system, to the morphism f which happened in our local system. This means that we have a functor that maps our theory into a local morphism for our experiment. This is depicted in figure 5.

We now see that there is an endofunctor of our local apparatus F and there is another functor, G, that takes the distant system and maps it into our local system. We have seen an example that seems to indicate that the endofunctor on our apparatus, the one that encodes an experiment, is, in fact, a monad. Thus, we have a local monad and a functor that maps a distant system to our local apparatus. There is a way to relate a monad with a suitably chosen functor into the local category (from another category) and we know this as the Adjoint Functor theorem. We depict the adjoint functor theorem in figure 7. This theorem states that every monad is the result of an adjunction. Thus, we now have a way of binding theories for a system under study to experiments which are done in a local lab.

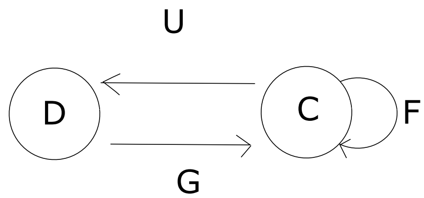

Figure 7: The Adjoint Functor Theorem

In particular the Adjoint Functor theorem states the following.

Given a monad on a category $\mathcal{C}$, $\{F, \mu, \eta\}$, there exists an adjunction which is a pair of functors between two categories. These functors map out of and into $\mathcal{C}$, relating to another category $\mathcal{D}$. So, we have three functors, G, U, and F.

$$F : \mathcal{C} \to \mathcal{C} \tag{2}$$

$$U : \mathcal{C} \to \mathcal{D} \tag{3}$$

$$G : \mathcal{D} \to \mathcal{C} \tag{4}$$

The functors U, G have a special relationship, in that there exists a family of bijections

$$hom_{\mathcal{C}}(UY, X) \cong hom_{\mathcal{D}}(Y, GX) \tag{5}$$

The theorem is quite abstract. We are going to need an example to show us what it means.

5.3 Radiation Shielding : An Example of the Adjoint Functor Theorem

Recall the experiment that showed the radiation shielding property of lead. We showed how the experiment was neatly encoded as a Monad. This monad, the List Mondad, must have several other features associated with it. The List monad should have a pair of functors G, U and a category, $\mathcal{D}$, that encodes our theory.

It is a well known fact that the List monad is the result of an adjunction between the category of Monoids and the category of Sets. Thus, the local category is SET, $\mathcal{C} = SET$. The theory category must be MON, the category of monoids, $\mathcal{D} = MON$. And the functors are called the free and forgetful functors between SET and MON.

For those who are unfamiliar, a Monoid is an algebraic structure. A moniod is very much like a group, except there is no built in notion of inverses. A model of the theory of Monoids in SET looks like this:

Definition 5.1. A Monoid consists of

1. A set, M, of monoid elements
2. A binary operation, $\cdot$,
3. An identity element, e, st. $\forall x \in M, e \cdot x = x \cdot e = x$

Where, $\forall x, y \in M, \exists z \in M s.t. x \cdot y = z$

Essentially, having a Monoid allows us to define a way to take two elements and combine them into a third element. The binary operation has an identity, e. An important example of a Monoid is $\mathbb{N}$, the Natural numbers. In the case of the Natural Numbers, the identity, e, is 0, and the binary operation is given by addition.

We must understand that the radiation shielding experiment is quite simplistic. It tells us very little about the system we are looking at. The only theory we have about our system under study is that radiation is made up of particles, which we call Alpha particles. As to their constituents and how they interact with matter (being the theory of electromagnetism, and the nuclear force) the experiment is not designed to speak about these things at all. We simply know that there is a number of particles hitting the detector every second. The number of particles is modeled as an element of the Natural numbers, $\mathbb{N}$. Recall that during the experiment, we introduced an increasing number of lead plates in the path of the radiation and this caused the numbers at the Geiger counter to change. The theory that explains this is quite simple: the number of Alpha particles reaching the Geiger counter has been changed. We see this depicted in figure 8.

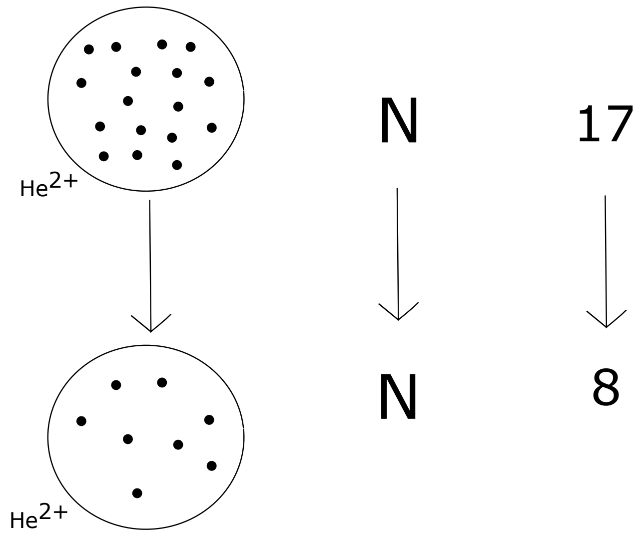

Figure 8: The change in the number of Alpha Particles is a Monoid map

From here we can begin to devise the natural isomorphism that ensures we have an adjunction. First, though, we need to describe the functors G, U. The functor G maps the category of Monoids into the Category of Sets and this functor is called the forgetful functor. It works by taking a monoid, $\mathcal{M}$ (which as we saw has a set M with some extra data) and returns nothing more than the set of elements, M. It essentially 'forgets' the extra structure. The functor U maps the category of Sets to the category of Monoids and is called the free functor. The free functor takes a Set, S, and gives something called the "free Monoid" on the set S. The free monoid is the simplest possible monoid and has no added structure other than the Monoid axioms given in the definition. The set of elements of the free monoid on S is the free product on the elements of S. That means that the elements of the free Monoid on S are all words made of elements of S, including the empty word. The elements of the free monoid are, colloquially, generated by the set S under the binary operation, $\cdot$.

The conceptual map is one that takes a change in the number of supposed alpha particles and maps it into the change in the lead plates and the Geiger counter. We see this in figure 9.

6 Future Directions

We have seen some interesting aspects of science reflected in the Monad axioms. Namely, we have seen how data for the same experiment, done by anyone, can be aggregated into a single body of information about the experiment. This was encoded in the axiom $\mu : F \cdot F \to F$. We have seen that the configurations of the apparatus are data too, and that is encoded in $\eta : 1_C \to F$. We also saw how the list Monad naturally encodes the fact that every measurement is done many times. This fact is deeper than the computation of probabilities, as those computations actually arise as a result of collecting many samples.

Two other important aspects of science should be encoded in our endofunctor F. First, we have to be able to copy the experiment. An axiom that could perform this could be $\upsilon : F \to F \cdot F$. Also, it is often the case that during an experiment we wish to discard information as it was bad in some way. An axiom that could reflect that would be $\tau : F \to 1_C$. It is a current conjecture that the true form of experiments should be Frobenius Monads, having all the axioms outlined along with the proper interaction between the Monad and Comonad natural transformations.

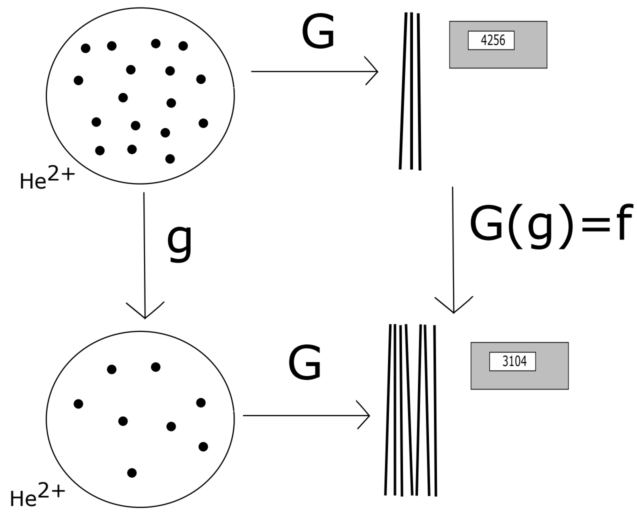

Figure 9: The change in the number of Alpha Particles maps to the change in the number of lead plates and readout of the counter

It would be fruitful to find a List-like Frobenius Monad and redo the reasoning stated here to see what kind of experiments one can model.

Though it has been a watershed of new insight, the categorical quantum mechanics has not led to a general derivation of quantum theory. Rather, it has been a kind of distillation of quantum properties into categorical properties. This paper attempts to establish epistemic notions that align perfectly with the evolution of a universe, making them, arguably, physically derived. The local category that we use as an apparatus actually represent nothing more than a local frame in a suitably generalized sense. This local category represents an epistemic restriction on knowing causal event structures from the past. Thus, like Spekkens' Toy Model [9], we derive a physical theory from a form of epistemic restriction, namely a restriction on knowing causal structure. Furthermore, the Monads that are used all live in a monoidal category where the monoidal product is given by functor composition. It would be interesting to know if the reasoning in this paper leads to a symmetric monoidal product with a dagger, all living in an endofunctor category. This could be seen as an actual derivation of the monoidal product from other, purely physical notions.

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