QUANTUM TOY MODEL: THE PACMAN VERSION

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Abstract. This paper attempts to expose the conceptual framework which an observer or scientist might use in order to develop a theory. We take Rob Spekkens' toy model as a starting point and use it to interpret a machine with a special type of probabilistic behaviour. By adjusting the machine, we introduce a doorway from his toy model to the real continuum of the Hilbert space and its operators. With our imaginary theorist's conceptual framework properly revealed, we expose some common interpretations of the ever important continuum and their effect on the interpretations of quantum theory.

1

There are several toy models which have been developed to display certain quantum-like behavior. One in particular is the toy model proposed in [Spekkens]. Is it possible to create a physical analog of this toy model? A physical model would have to obey the basic premise that an observer can receive answers to only half of the questions which are required to specify the ontic state of the system.

The physical system proposed here centers around a type of coin flip (fig. **??**) in which, the results of the flip can take one of four possible states. The coin is more of an almond or football shaped object, red on the top, blue on the bottom with a single arrow on either side, each pointing in the same direction.

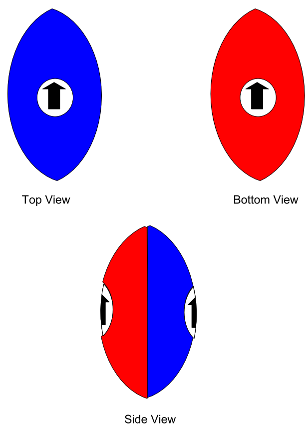

Figure 1. A coin with 4 states

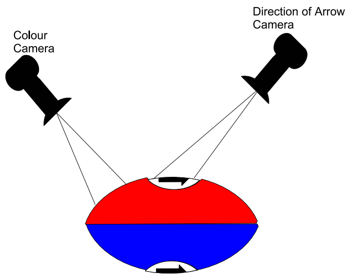

FIGURE 2. Each camera gives an answer to only one of two questions

The idea is that the almond shaped coin fits into an almond shaped hole so that only one side is pointing up and visible after the coin is tossed.

Trained on the almond are two cameras (fig.**??**). One camera is focused on the arrow, giving no clue as to which colour is visible. The other camera is trained on a coloured spot, without giving away the direction of the arrow. The four ontic states of the coin are

- Red side up, arrow pointing North
- Red side up, arrow pointing South
- Blue side up, arrow pointing North
- Blue side up, arrow pointing South

This kind of coin is chosen only to provide 4 ontic states as in [Spekkens]. These, of course, are chosen to mimic a qubit. One could just as easily put two independent coins in the machine and label one $coinA$ and the other $coinB$. The outputs would be something like H_1, T_1, H_2, T_2. It is the epistemic states which one defines that are of significance and they don't reflect much more than one's interpretation of the machine. They don't reflect an aspect of reality which we would say is the ontic state of the two coins, they being HH, TT, HT, TT.

Returning to our coin and cameras, we complete the machine. The whole apparatus is encased in a box with a viewing screen and some buttons (fig. **??**).

There are three buttons on the machine, $\{Run, A, B\}$. The Run button is meant to flip the coin and take a photo from only one of the two cameras. The buttons A and B chose which camera the photo will be taken from. One and only one of the $\{A, B\}$ can be selected before the Run button will activate the coin toss and the photo. Finally, the state of the screen only changes when the Run button is pressed.

One of four things may appear on the screen: a red screen, a blue screen, an arrow pointing up, and an arrow pointing down (fig. **??**). These correspond to four possible answers to some pair of abstract yes-no questions.

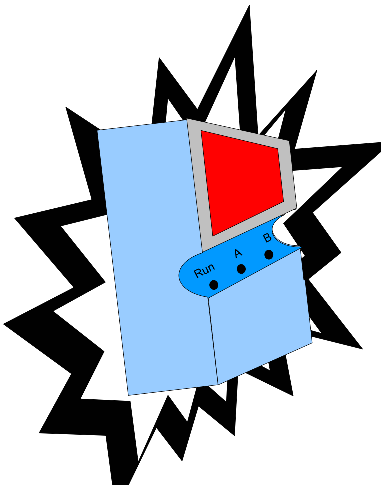

FIGURE 3. The machine

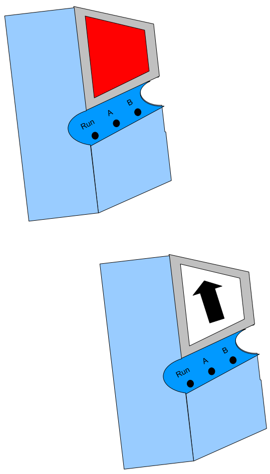

FIGURE 4. Only one answer is displayed for each run

2. The Epistemic States

One may point out that the existence of such a physical devices is, really, not very interesting. That is, until one begins to define epistemic states as in [Spekkens].

Recall that the ontic states are

- Red side up, arrow pointing North
- Red side up, arrow pointing South
- Blue side up, arrow pointing North
- Blue side up, arrow pointing South

The epistemic states are taken directly from [Spekkens]

$$1 \vee 2$$
$$1 \vee 3$$
$$1 \vee 4$$
$$2 \vee 3$$
$$2 \vee 4$$
$$3 \vee 4$$

In **??** we see that he develops an algebraic structure by way of some axioms. We see this also in CBH, and in Hardy. These are all attempts to derive, from a set of axioms, an algebraic structure which is the Hilbert space and the Lie group of its operators . One can more easily present that structure in purely algebraic terms. The axioms are an attempt to associate some sort of meaning to the structures, but do not provide the only means to present the structures. Furthermore, once an abstract structure is outlined, there are numerous representations of that structure, from matrices to braids.

3. What's the Big Deal?

This paper owes everything to **??**, really. I don't think there is anything interesting about the existence of a machine which obeys the properties which **??** outlines. However, that is not to say that this machine is not at all uninteresting.

If one thinks, for a moment, about a Stern Gerlach experiment, we could potentially put one inside our machine and no one would be the wiser. We could take operators σ_x and σ_x and assign the four screen states to the basis vectors of those operators. The $\{A, B\}$ buttons would then chose either σ_x or σ_y and with some programming we could assign the red screen to show up when if $|0\rangle$ appears, blue if $|1\rangle$ appears etc.

For a moment, lets say that we never told our observer that we performed the switch. In fact, for a moment, let us suppose that we never told our unfortunate observer anything. Not even classical probability theory. What is our unfortunate subject to do when encountering this odd machine?

From his perspective, he has some buttons to press and a button that seems to initiate something, or at least returns something he may interpret as information. This begs the question, why does he think that the screen is conveying information? We could use strange symbols instead of colours and arrows. Were he to find a single strange symbol, he may infer any number of things as to the meaning of the symbol.

Without any model for what is going on inside the machine, how does our guest begin to develop a theory? How could he develop a probability theory? Without a model, how can he say what the instrumental states are, much less the ontic states? He does not know if another colour, simply very unlikely, will appear some time in the future. He does not know if the arrow may point sideways at some point.

I think the merit of the machine is simply to strip away our usual starting points for developing theories, including toy theories. It begs certain questions about what our theories have to do with reality. I think.

One other interesting thing about our machine is the fact that, while it may present a physical analog of [**??**] Toy model, it does not represent interactions with a quantum device. Recall that the toy model in [**??**] did not reproduce quantum theory, just something very much like it. It seems that the discrete program is at heart here. The model does not produce something called a $C*$ category which, to me, means that it does not produce some algebra with manifold structure. It is possibly the existence of a Boolean algebra on the topology of the manifold which gives Quantum theory and also gives quantum theory its background dependance.

All that conjecture aside, what if a dial were added to our machine? We can ask ourselves anew what our guest might imagine is going on inside the machine. Perhaps, by starting with nothing but our machine, and by suggesting it motivates a theory such as that proposed in [**??**] we can then ask ourselves how the dial motivates our observer. Say for the moment that the inclusion of the dial actually generates quantum theory, simply because it introduces that all important continuum into the theories of our observer. An argument as to why that dial motivates our observer, and why the machine itself motivates our observer...is well, just interesting.

The most important point of this essay is to force the reader into a position in which they must identify the framework which they are using to interpret the so called *information* which is coming out of the machine, both from the screen and the use of the buttons. By keeping this in mind, the addition of the dial to the machine will force us to identify how we see the dial. The dial simply represents the continuum. We will be forced to understand how we are concieving the continuum as we explain the conceptual movement from the Sprott machine (no longer the Spekkens machine) to the quantum theory. We are going to have to devise an epistemic state interpretation for the dial.

References

1. R. W. Spekkens, quant-ph/0401052
2. . Rovelli arXiv:quant-ph/9609002 v2 24 Feb 1997

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